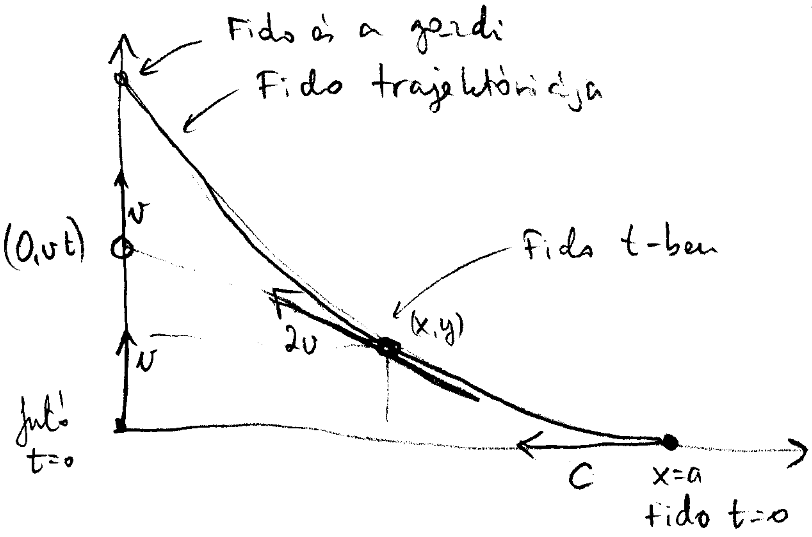
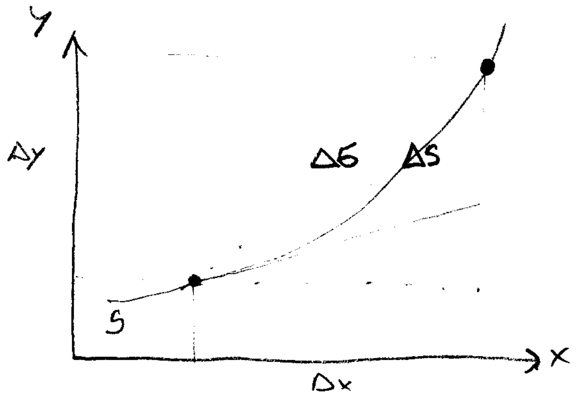
Fidó és a garda

Probléma:

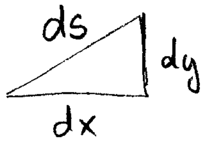
Fido seb.: c
Gardi seb.: v



Differenciál egy görbe mentén:



$$\boxed{\frac{ds}{dx} = \lim_{\Delta x \to 0} \frac{\Delta s}{\Delta x} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$



$$ds^2 = dx^2 + dy^2$$

$$\left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\frac{ds}{dx} = \sqrt{1+(y')^2}$$

Görbe hossza:

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$s = \int_a^b ds = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \sqrt{dx^2+dy^2}$$

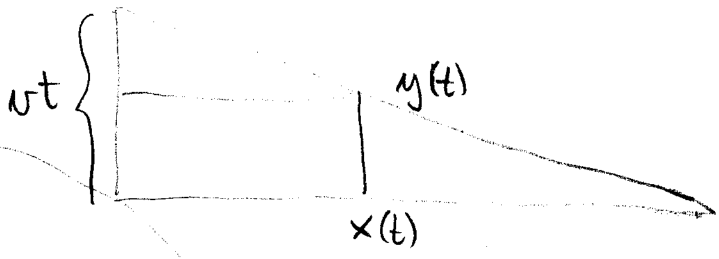
Mi van, ha y és x egy t paraméteren keresztül függenek egymástól

$$x = x(t), \quad y = y(t)$$

$$s = \int_a^b \sqrt{dx^2+dy^2} = \int_{t_a}^{t_b} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$$

Megoldás

Futó : $y = vt$



Fidó : $\dfrac{dy}{dx} = -\dfrac{vt - y(t)}{x(t)}$

Megtett út : $s = ct \Rightarrow t = \dfrac{s}{c}$

$$x \cdot \frac{dy}{dx} = y - \frac{sv}{c} \Rightarrow s = \frac{c}{v}\left(y - x\frac{dy}{dx}\right) \Rightarrow$$

$$\frac{ds}{dx} = \frac{c}{v}\left(\frac{dy}{dx} - \frac{dy}{dx} - x\frac{d^2y}{dx^2}\right) = -\frac{c}{v}\, x \,\frac{d^2y}{dx^2} = -\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$-\frac{c}{v}\, x\, \frac{dy'}{dx} = -\sqrt{1+(y')^2}$$

$$\boxed{\frac{dy'}{\sqrt{1+(y')^2}} = \frac{v}{c}\,\frac{dx}{x}}$$

diff. egyenlet integráljuk

Alapintegrál

$$\boxed{\ln\left(y' + \sqrt{1+(y')^2}\right) + C = \frac{v}{c}\ln x}$$

Peremfeltétel

$t = 0 \quad y' = 0$ (Fidó x tengely mentén fut)

$x = a$

$$C = \frac{v}{c}\ln a = \ln a^{v/c}$$

$$\ln\left[\left(y' + \sqrt{1+y^2}\right)\cdot a^{v/c}\right] = \ln x^{v/c}$$

$$y' + \sqrt{1+y'^2} = \left(\frac{x}{a}\right)^{v/c} \qquad (1)$$

$$\left(y' + \sqrt{1+y'^2}\right)\left(y' - \sqrt{1+y'^2}\right) = \left(\frac{x}{a}\right)^{v/c} \cdot \left(y' - \sqrt{1+y'^2}\right) = -1 \quad (3)$$

$$\cancel{(y')^2} - 1 \cancel{-(y')^2} \qquad\qquad y' - \sqrt{1+y'^2} = -\left(\frac{a}{x}\right)^{v/c} \quad (2)$$

$(1)+(2) =$

$$\Rightarrow 2y' = \left(\frac{x}{a}\right)^{v/c} - \left(\frac{a}{x}\right)^{v/c} \Longrightarrow$$

$$\boxed{y = \frac{1}{2}\frac{1}{a^{v/c}} \cdot \frac{x^{v/c+1}}{v/c+1} - \frac{1}{2} a^{v/c} \cdot \frac{x^{-v/c+1}}{-v/c+1} + c_2}$$

Peremfeltétel: $\quad x = a \;,\; y = 0$

$$0 = \frac{1}{2}\frac{a^{\cancel{v/c}+1}}{\cancel{a^{v/c}}}\frac{1}{v/c+1} - \frac{1}{2} \cancel{a^{v/c}} \cdot a^{\cancel{-v/c}+1}\frac{1}{-v/c+1} + c_2$$

$$\frac{a}{v/c+1} - \frac{a}{-v/c+1} = -2c_2$$

Ha $\underline{\underline{c = 2v}}$

$$2c_2 = -\frac{a}{\frac{1}{2}+1} + \frac{a}{-\frac{1}{2}+1} = -\frac{a}{3/2} + \frac{a}{1/2} = -\frac{2}{3}a + 2a = a\frac{-2+6}{3} = \frac{4}{3}a$$

$$c_2 = \frac{2a}{3} \qquad\qquad v/c = 1/2$$

$$y = \frac{1}{2}\frac{1}{\sqrt{a}}\frac{x^{3/2}}{3/2} - \frac{1}{2}\sqrt{a}\frac{x^{1/2}}{1/2} + \frac{2a}{3} =$$

$$\boxed{y = \frac{x^{3/2}}{3\sqrt{a}} - \sqrt{ax} + \frac{2a}{3}}$$

Fido pályája

Találkozás a gazdival

$x=0$

$y=\frac{2a}{3}$ gazda által megtett út y tengely mentén

$t=\frac{2a}{3v}$ idő alatt

Fido által megtett út: $\frac{4a}{3}$ (kétszer annyi)