$F(k) = \int_{-\infty}^{\infty} f(x) K(k,x)\, dx$ az f(x) fgv. integráltranszformáltja, (1)

$K(k,x)$ a kernel (mag)

Laplace-transzformáció

$$F(s) = \mathcal{L} f(t) = \int_0^\infty e^{-st} f(t)\, dt$$

Pl.:

1) $f(t) = 1$

$$\underline{\underline{\mathcal{L}(1)}} = \int_0^\infty e^{-st}\, dt = \left[-\frac{1}{s} e^{-st}\right]_0^\infty = 0 + \frac{1}{s} = \underline{\underline{\frac{1}{s}}} \qquad t>0$$

2) $f(t) = e^{kt}$

$$\underline{\underline{\mathcal{L}(e^{kt})}} = \int_0^\infty e^{kt} e^{-st}\, dt = \int_0^\infty e^{-(s-k)t}\, dt = \underline{\underline{\frac{1}{s-k}}} \qquad s>k$$

3) $f(t) = \cosh(kt) = \frac{1}{2}\left(e^{kt} + e^{-kt}\right)$

$$\underline{\underline{\mathcal{L}(\cosh(kt))}} = \frac{1}{2}\left(\mathcal{L}(e^{kt}) + \mathcal{L}(e^{-kt})\right) = \frac{1}{2}\left(\frac{1}{s-k} + \frac{1}{s+k}\right) =$$

$$= \frac{1}{2}\,\frac{s+k+s-k}{s^2-k^2} = \underline{\underline{\frac{s}{s^2-k^2}}}$$

Hasonlóan $\underline{\underline{\mathcal{L}(\sinh(kt))}} = \underline{\underline{\frac{k}{s^2-k^2}}}$

4) $e^{ikt} = \cos kt + i \sin kt$

$$\mathcal{L}(e^{ikt}) = \mathcal{L}(\cos kt) + i\,\mathcal{L}(\sin kt) = \frac{1}{s-ik}$$

$$\sin kt = \frac{1}{2i}\left(e^{ikt} - e^{-ikt}\right)$$

$$\underline{\mathcal{L}(\sin kt)} = \frac{1}{2i}\left(\mathcal{L}(e^{ikt}) - \mathcal{L}(e^{-ikt})\right) = \frac{1}{2i}\left(\frac{1}{s-ik} - \frac{1}{s+ik}\right) =$$

$$= \frac{1}{2i}\,\frac{s+ik-(s-ik)}{s^2+k^2} = \frac{1}{2i}\,\frac{2ik}{s^2+k^2} = \underline{\underline{\frac{k}{s^2+k^2}}}$$

$$\mathcal{L}(\cos kt) = \mathcal{L}\left(\frac{1}{2}\left(e^{ikt} + e^{-ikt}\right)\right) = \frac{1}{2}\left(\frac{1}{s-ik} + \frac{1}{s+ik}\right) =$$

$$= \frac{1}{2} \frac{s+ik+s-ik}{s^2+k^2} = \frac{s}{s^2+k^2}$$

$$e^{ikt} = \cos kt + i \sin kt$$
$$e^{-ikt} = \cos(-kt) + i \sin(-kt)$$

$$e^{ikt} = \cos kt + i \sin kt$$
$$e^{-ikt} = \cos kt - i \sin kt$$
$+$

$$\cos kt = \frac{e^{ikt} + e^{-ikt}}{2}$$

$$\sin kt = \frac{e^{ikt} - e^{-ikt}}{2i}$$

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5) $f(t) = t^n$

$$\mathcal{L}(t^n) = \int_0^\infty e^{-st} t^n \, dt = \frac{\Gamma(n+1)}{s^{n+1}} = \frac{n!}{s^{n+1}}$$

$\Gamma(x)$ gamma - fgv :

$$\Gamma(x) = \begin{cases} \int_0^\infty e^{-t} t^{x-1} \, dt & \text{Euler-féle integrál} \quad x > 0 \\ \lim_{n\to\infty} \frac{n! \, n^{x-1}}{x(x+1)(x+2)\cdots(x+n-1)} & \forall x \end{cases}$$

$$\Gamma(x+1) = \Gamma(x) \cdot x \qquad \Gamma(x) \cdot \Gamma(1-x) = \frac{\pi}{\sin \pi x}$$

$\Gamma(n) = (n-1)!$ ha $n > 0$ egész szám

$n = 1$:

6). $F(s-a) = \int_0^\infty e^{-(s-a)t} f(t)\, dt = \mathcal{L}[e^{at} f(t)]$ (3)

Pl.:

$$\boxed{\begin{aligned} \mathcal{L}(e^{at}\cos kt) &= \frac{s-a}{(s-a)^2+k^2} \\ \mathcal{L}(e^{at}\sin kt) &= \frac{k}{(s-a)^2+k^2} \end{aligned}}$$

$\boxed{\text{Eltolási szabály}}$

Derivatives:

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = \int_0^\infty \underset{v}{e^{-st}} \underset{u'}{\frac{df(t)}{dt}}\, dt =$$

$$= \left[\underset{u}{f(t)}\, \underset{v}{e^{-st}}\right]_0^\infty - \int_0^\infty \underset{v'}{(-s \cdot e^{-st})} \cdot \underset{u}{f(t)}\, dt =$$

$$= -f(0) + s \cdot \int_0^\infty e^{-st} f(t)\, dt = -f(0) + s \cdot \mathcal{L}(f(t))$$

Hasonlóan:

$$\mathcal{L}\left[\frac{d^2 f(t)}{dt^2}\right] = s^2 \mathcal{L}(f(t)) - s f(0) - f'(0)$$

Pl.: $\sin kt$

$$\frac{d^2 \sin kt}{dt^2} = -k^2 \sin kt$$

$$\mathcal{L}\left(\frac{d^2 \sin kt}{dt^2}\right) = -k^2 \mathcal{L}(\sin kt)$$

$$= s^2 \mathcal{L}(\sin kt) - \underset{\emptyset}{\sin(0)} - \underset{\substack{k\cos kt \\ k}}{\left.\frac{d \sin kt}{dt}\right|_0}$$

$$\mathcal{L}(\sin kt) = \frac{k}{s^2+k^2}$$

Differenciálegyenletek:

Harmonikus oszcillátor:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad X(s) = \mathcal{L}(x(t))$$

$$\mathcal{L}(x'') + \frac{k}{m}\mathcal{L}(x) = 0$$

Kezdőérték: $x(0) = x_0$, $x'(0) = 0$

$$s^2X(s) - sx_0 - 0 + \frac{k}{m}X(s) = 0$$

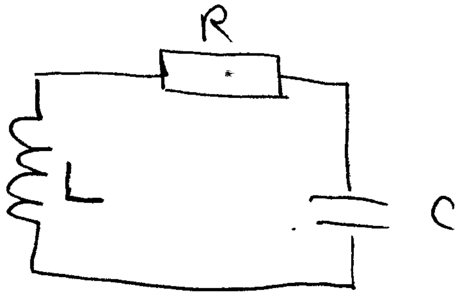
$$X(s) = \frac{s}{s^2 + (k/m)^2}\,x_0 \qquad k \equiv \frac{k}{m}$$

Látható: $\mathcal{L}(\cos kt) = \frac{s}{s^2+k^2}$

$$x(t) = x_0 \cdot \cos\frac{k}{m}t$$

Csillapított oszcillátor

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq(t)}{dt} + \frac{1}{LC}q(t) = 0$$



$$q(0) = q_0, \quad \left.\frac{dq}{dt}\right|_0 = 0$$

$$\mathcal{L}(q'') + \frac{R}{L}\mathcal{L}(q') + \frac{1}{LC}\mathcal{L}(q) = 0 \qquad X(s) = \mathcal{L}(q)$$

$$\mathcal{L}(q') = s\,\mathcal{L}(q) - q_0 = s\,X(s) - q_0$$

$$s^2X(s) + \frac{R}{L}\left(\alpha X(s) - q_0\right) + \frac{1}{LC}X(s) = 0$$

$$X(s) = q_0 \cdot \frac{s + R/L}{s^2 + s(R/L) + 1/LC}$$

⑤

$$X(s) = q_0 \frac{s + \frac{R}{L}}{\left(s+\frac{R}{2L}\right)^2 + \omega_1^2} =$$

$$\omega_1^2 = \frac{1}{LC} - \frac{R^2}{4L^2} > 0$$

$$\mathcal{L}\left(e^{at}\cos\omega t\right) = \frac{s-a}{(s-a)^2+\omega^2}$$

$$\mathcal{L}(\cos\omega t) = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}(\sin\omega t) = \frac{\omega}{s^2+\omega^2}$$

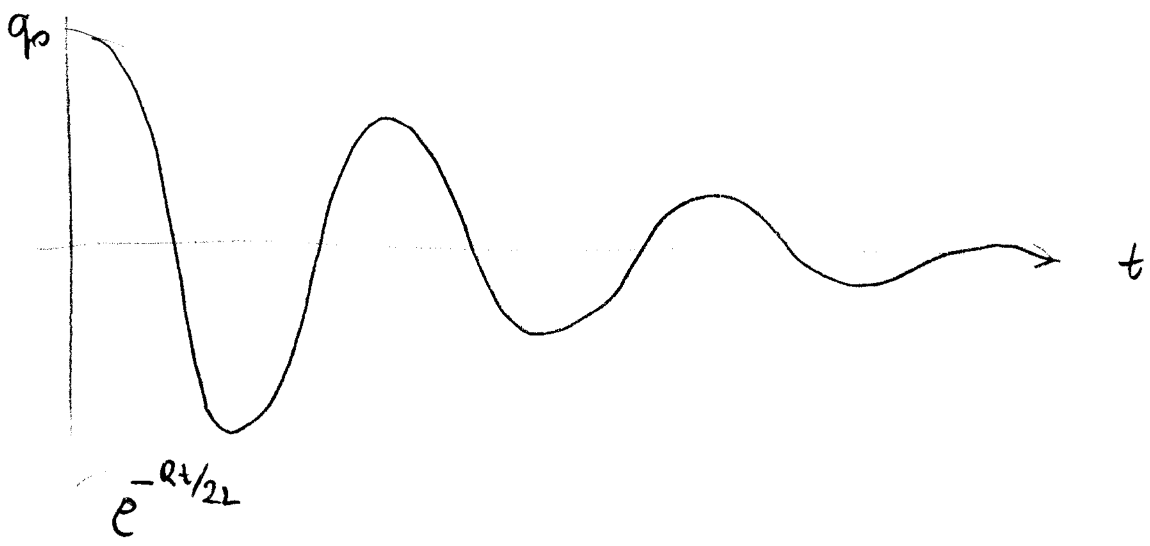
$$\mathcal{L}\left(e^{at}\sin\omega t\right) = \frac{\omega}{(s-a)^2+\omega^2}$$

$$= q_0 \frac{s+\frac{R}{2L}}{\left(s+\frac{R}{2L}\right)^2+\omega^2} + q_0 \frac{\frac{R}{2L}}{\left(s+\frac{R}{2L}\right)^2+\omega^2} =$$

$$= q_0 \frac{s+\frac{R}{2L}}{\left(s+\frac{R}{2L}\right)^2+\omega^2} + q_0 \frac{R}{2L\omega} \frac{\omega}{\left(s+\frac{R}{2L}\right)^2+\omega^2} =$$

$$\mathcal{L}\left(e^{-\frac{R}{2L}} \cdot \cos\omega t\right) \qquad \mathcal{L}\left(e^{-\frac{R}{2L}} \sin\omega t\right)$$

$$= q_0\, e^{-Rt/2L}\left(\cos\omega t + \frac{R}{2L\omega}\sin\omega t\right)$$



Csillapított rezgő mozgás

$$\ddot{x} + \frac{h}{m}\dot{x} + \frac{K}{m}x = 0$$

Helyettesítés: $x(t) = z(t)\, e^{\lambda t}$

$$e^{\lambda t}\underbrace{\left[\ddot{z} + \left(2\lambda + \frac{h}{m}\right)\dot{z} + \left(\lambda^2 + \frac{h\lambda}{m} + \frac{K}{m}\right) z\right]}_{\emptyset} = 0$$

λ választható, hogy $\left(2\lambda + \frac{h}{m}\right)$ eltűnik.

$$\lambda = -h/2m$$

$$\ddot{z} + \left(\frac{K}{m} - \frac{h^2}{4m^2}\right) z = 0$$

Megoldás z-re: $z = C \cdot \cos \omega_1 t + D \sin \omega_1 t$

ahol: $\omega_1^2 = \frac{K}{m} - \frac{h^2}{4m^2}$

Megoldás x-re:

$$x = e^{-(h/2m)t}\left[C \cdot \cos \omega_1 t + D \cdot \sin \omega_1 t\right]$$