Appendix II: Dimensions and Units

The numerical values of most physical quantities are expressed in terms of units. The distance between two points, for example, can be specified by the number of meters (or feet, Ångströms, *etc.*). Similarly, time can be expressed in seconds, days or, say, years. However, the number of days per year varies from one year to another. The quantities, distance (length) and time, as well as mass, are usually chosen to be primary quantities. In terms of them Newton's second law for the force on an object, can be written as force = mass.distance/(time)^2. The definition of the primary quantities allows dimensional expressions to be written, such as [force] = $\mathrm{MLT^{-2}}$ in the present example. Note, however, that in everyday life one speaks of the weight of an object (or a person). Of course the weight is not the mass, but rather the force acting on the object by the acceleration due to gravity: [acceleration] = $\mathrm{LT^{-2}}$.

The dimensional expressions given above are determined by the particular choice of primary quantities. In the international system (SI) the base units of mass, length and time have been chosen to be kilogram, meter and second, respectively. Then, the newton (N, a derived unit) is the unit of force. In the cgs system the centimeter, gram and second are considered to be the base units, leading to the force expressed in dynes. In this particular example the dimensional equation [force] = $\mathrm{MLT^{-2}}$ applies to either choice of units. As a force that produces a change in distance involves work or energy, their dimensions are given by $\mathrm{ML^2T^{-2}}$. The unit of energy in SI is the joule (J), while in cgs it is the erg (note that 1 erg = 10^{-7} J).

The primary quantities M, L, T are sufficient to describe most problems in mechanics. In thermodynamics and other thermal applications it is customary to add an absolute temperature. In this case the dimension of the Boltzmann constant, for example, is given by $[k] = \mathrm{ML^2T^{-2}}\theta^{-1}$, where the symbol θ is used here for the dimension of the absolute or thermodynamic temperature.

The dimensions of units in electricity and magnetism are the origin of much confusion. In the days when mechanical and thermal quantities were expressed in cgs, two different systems were introduced for the electrical and magnetic quantities. They are the esu (electrostatic units) and the emu (electromagnetic units), respectively. Their addition to the cgs system results in a hybrid that is usually referred to as the Gaussian system. An apparent advantage of the Gaussian system is the disappearance of the factor $4\pi\varepsilon_0$ which is forever present in SI in problems involving inherent spherical symmetry. On the other hand, in the Gaussian system a given quantity usually has different values in esu and emu. It then becomes necessary to introduce various powers of the velocity of light (c) to assure internal consistency.

The so-called atomic units are often employed in quantum mechanical calculations. They are combinations of fundamental constants that are treated as if they were units. The base dimensions are chosen to be mass, length, charge and action. They are respectively the rest mass of the electron (m_e), the radius of the first Bohr orbit ($a_0 = 4\pi\varepsilon_0\hbar^2/m_e e^2$), the elementary charge (e) and the action ($\hbar = h/2\pi$, where h is Planck's constant). The corresponding energy, given by $E_h = \hbar^2/m_e a_0^2 = m_e e^4/(4\pi\varepsilon_0)^2\hbar^2$, is expressed in hartree.

The following tables summarize the units used in this book. For more extensive tabulations, the reader is referred to the "Green Book", Ian Mills, *et al.* (eds), "Quantities, Units and Symbols in Physical Chemistry", Blackwell Scientific Publications, London (1993).

Table 1 The SI base units.

Physical quantity (dimension in SI)	SI unit	Symbol
length (L)	meter	m
mass (M)	kilogram	kg
time (T)	second	s
thermodynamic or absolute temperature (θ)	kelvin	K
electric current (A)	ampere	A
amount of substance (mol)	mole	mol
luminous intensity (cd)	candela	cd

Table 2 Some derived units.

Physical quantity	SI unit	Symbol	Expression in SI	Dimension in SI
frequency	hertz	Hz	s^{-1}	T^{-1}
force	newton	N	$m\ kg\ s^{-2}$	$M\ L\ T^{-2}$
pressure	pascal	Pa	$N\ m^{-2}$	$M\ L^{-1}T^{-2}$
energy, work	joule	J	N m	$M\ L^2T^{-2}$
power	watt	W	$J\ s^{-1}$	$M\ L^2T^{-3}$
electric charge	coulomb	C	A s	A T
electric potential (emf)	volt	V	$J\ C^{-1}$	$M\ L^2T^{-3}A^{-1}$
electric resistance	ohm	Ω	$V\ A^{-1}$	$M\ L^2T^{-3}A^{-2}$
electric capacitance	farad	F	$C\ V^{-1}$	$M^{-1}L^{-2}T^4A^2$

Table 2 (*Continued*).

Physical quantity	SI unit	Symbol	Expression in SI	Dimension in SI
electric field strength			$V\ m^{-1}$	$L^2T^{-3}A^{-1}$
Celsius temperature	degree Celsius	C	K	θ
density			$kg\ m^{-3}$	$M\ L^{-3}$
molar volume			m^3mol^{-1}	L^3mol^{-1}
wavenumber[a]			m^{-1}	L^{-1}
permittivity (vacuum)		$\varepsilon(\varepsilon_0)$	$F\ m^{-1}$	$M^{-2}L^{-2}T^4A^2$

[a] Wavenumber is invariably expressed in cm^{-1}.

Table 3 Prefixes in SI.

Submultiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-1}	deci	d	10	deca	da
10^{-2}	centi	c	10^2	hecto	h
10^{-3}	milli	m	10^3	kilo	k
10^{-6}	micro	μ	10^6	mega	M
10^{-9}	nano	n	10^9	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E

Table 4 Some of the fundamental constants in the SI system.

Quantity	Symbol	Value
permeability of vacuum	μ_0	$4\pi 10^{-7}H\ m^{-1}$ or $V\ s\ A^{-1}m^{-1}$ (defined)
permittivity of vacuum	$\varepsilon_0 = (m_0c^2)^{-1}$	$8.854\ 187\ 816\ldots 10^{-12}F\ m^{-1}$ or $C^2J^{-1}m^{-1}$
speed of light in vacuum	c_0	$299\ 792\ 458\ m\ s^{-1}$ (defined)
Planck constant	h	$6.626\ 075\ 5(40)\ 10^{-34}\ J\ s$
elementary charge	e	$1.602\ 177\ 33(49)\ 10^{-19}\ C$
electron rest mass	m_e	$9.109\ 389\ 7(54)\ 10^{-31}\ kg$
proton rest mass	m_p	$1.672\ 623\ 1(10)\ 10^{-27}\ kg$
Avogadro constant	N_A	$6.022\ 136\ 7(36)\ 10^{23}mol^{-1}$
Boltzmann constant	k	$1.380\ 658\ (12)\ 10^{-23}J\ K^{-1}$
gas constant	R	$8.314\ 510\ (70)\ J\ K^{-1}mol^{-1}$
zero of the Celsius scale		273.15 K (defined)
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2/m_ee^2$	$5.291\ 772\ 49(24)\ 10^{-11}\ m$
Hartree energy	$E_h = \hbar^2/m_ea_0^2$	$4.359\ 748\ 2(26)\ 10^{-18}\ J$
Rydberg constant	$R_\infty = E_h/2hc_0$	$1.097\ 373\ 153\ 4(13)\ 10^7m^{-1}$