Appendix VIII: Character Tables for Some of the More Common Point Groups

Character tables are given in this appendix for the following point groups: (1) the groups which correspond to the 32 crystal classes (see Table 8-14), (2) groups containing 5-fold axes that may be needed to describe the symmetries of certain molecules or complex ions, and (3) the infinite groups $\mathcal{C}_{\infty v}$ and $\mathcal{D}_{\infty h}$ that are appropriate to linear structures.

Some comment on the notation for the irreducible representations (symmetry species) is necessary, as certain variations will be found in the literature. Molecular spectroscopists usually designate one-dimensional symmetry species by A or B, two-dimensional species by E, and three-dimensional species by F or T. Some authors prefer lower-case letters to capitals. The symmetry or antisymmetry of a given species with respect to the generating rotation operation distinguishes A and B. Subscripts 1 and 2 are used on A and B according to the symmetry with respect to rotation about a C_2 axis perpendicular to the generating axis. Symmetry with respect to inversion is indicated by a subscript g or u (see Section 4.4.2), while a prime or double prime identifies species that are, respectively, symmetric or antisymmetric with respect to a horizontal plane. Finally numerical subscripts are used to distinguish various doubly and triply degenerate species.

For linear molecules or ions the symbols are usually those derived from the term symbols for the electronic states of diatomic and other linear molecules. A capital Greek letter $\Sigma, \Pi, \Delta, \Phi, \ldots$ is used, corresponding to $\lambda = 0, 1, 2, 3, \ldots$, where λ is the quantum number for rotation about the molecular axis. For Σ species a superscript + or − is added to indicate the symmetry with respect to a plane that contains the molecular axis.

The components of the translation and rotation vectors are given as T_x, T_y, T_z and R_x, R_y, R_z, respectively. The components of the polarizability tensor appear as linear combinations such as $\alpha_{xx} + \alpha_{yy}$, *etc.*, that have the symmetry of the indicated irreducible representation.

Table 1 Character tables for the cyclic groups, $\mathcal{C}_n (n = 2, 3, 4, 5, 6)$.

$\mathcal{C}_2$	E	C_2		
A	1	1	T_z, R_z	$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}, \alpha_{xy}$
B	1	−1	T_x, T_y, R_x, R_y	α_{yz}, α_{zx}

$\mathcal{C}_3$	E	C_3	C_3^2		$\varepsilon = exp(2\pi i/3)$
A	1	1	1	T_z, R_z	$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
E	$\left\{\begin{matrix} 1 \\ 1 \end{matrix}\right.$	$\begin{matrix} \varepsilon \\ \varepsilon^\star \end{matrix}$	$\left.\begin{matrix} \varepsilon^\star \\ \varepsilon \end{matrix}\right\}$	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy}), (\alpha_{yz}, \alpha_{zx})$

$\mathcal{C}_4$	E	C_4	C_2	C_4^3		
A	1	1	1	1	T_z, R_z	$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
B	1	−1	1	−1		$\alpha_{xx} - \alpha_{yy}, \alpha_{zz}$
E	$\left\{\begin{matrix} 1 \\ 1 \end{matrix}\right.$	$\begin{matrix} i \\ -i \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\left.\begin{matrix} -i \\ i \end{matrix}\right\}$	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$

$\mathcal{C}_5$	E	C_5	C_5^2	C_5^3	C_5^4		$\varepsilon = exp(2\pi i/5)$
A	1	1	1	1	1	T_z, R_z	$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
E_1	$\left\{\begin{matrix} 1 \\ 1 \end{matrix}\right.$	$\begin{matrix} \varepsilon \\ \varepsilon^\star \end{matrix}$	$\begin{matrix} \varepsilon^2 \\ \varepsilon^{2\star} \end{matrix}$	$\begin{matrix} \varepsilon^{2\star} \\ \varepsilon^2 \end{matrix}$	$\left.\begin{matrix} \varepsilon^\star \\ \varepsilon \end{matrix}\right\}$	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$
E_2	$\left\{\begin{matrix} 1 \\ 1 \end{matrix}\right.$	$\begin{matrix} \varepsilon^2 \\ \varepsilon^{2\star} \end{matrix}$	$\begin{matrix} \varepsilon^\star \\ \varepsilon \end{matrix}$	$\begin{matrix} \varepsilon \\ \varepsilon^\star \end{matrix}$	$\left.\begin{matrix} \varepsilon^{2\star} \\ \varepsilon^2 \end{matrix}\right\}$		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$

$\mathcal{C}_6$	E	C_6	C_3	C_2	C_3^2	C_6^5		$\varepsilon = exp(2\pi i/6)$
A	1	1	1	1	1	1	T_z, R_z	$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
B	1	−1	1	−1	1	−1		
E_1	$\left\{\begin{matrix} 1 \\ 1 \end{matrix}\right.$	$\begin{matrix} \varepsilon \\ \varepsilon^\star \end{matrix}$	$\begin{matrix} -\varepsilon^\star \\ -\varepsilon \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\begin{matrix} -\varepsilon \\ -\varepsilon^\star \end{matrix}$	$\left.\begin{matrix} \varepsilon^\star \\ \varepsilon \end{matrix}\right\}$	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz} - \alpha_{zx})$
E_2	$\left\{\begin{matrix} 1 \\ 1 \end{matrix}\right.$	$\begin{matrix} -\varepsilon^\star \\ -\varepsilon \end{matrix}$	$\begin{matrix} -\varepsilon \\ -\varepsilon^\star \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} -\varepsilon^\star \\ -\varepsilon \end{matrix}$	$\left.\begin{matrix} -\varepsilon \\ -\varepsilon^\star \end{matrix}\right\}$		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$

Table 2 Character tables for the dihedral groups, $\mathscr{D}_n (n = 2, 3, 4, 5, 6)$.

$\mathscr{D}_2 = \mathscr{V}$	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1		$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}$
B_1	1	1	−1	−1	T_z, R_z	α_{xy}
B_2	1	−1	1	−1	T_y, R_y	α_{zx}
B_3	1	−1	−1	1	T_x, R_x	α_{yz}

$\mathscr{D}_3$	E	$2C_3$	$3C_2'$		
A_1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2	1	1	−1	T_z, R_z	
E	2	−1	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy}), (\alpha_{yz}, \alpha_{zx})$

$\mathscr{D}_4$	E	$2C_4$	C_2	$2C_2'$	$2C_2''$		
A_1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	−1	−1	T_z, R_z	
B_1	1	−1	1	1	−1		$\alpha_{xx} - \alpha_{yy}$
B_2	1	−1	1	−1	1		α_{xy}
E	2	0	−2	0	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$

$\mathscr{D}_5$	E	$2C_5$	$2C_5^2$	$5C_2'$		$\alpha = 72°$
A_1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	−1	T_z, R_z	
E_1	2	$2\cos\alpha$	$2\cos 2\alpha$	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$
E_2	2	$2\cos 2\alpha$	$2\cos\alpha$	0		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$

$\mathscr{D}_6$	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$		
A_1	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	1	−1	−1	T_z, R_z	
B_1	1	−1	1	−1	1	−1		
B_2	1	−1	1	−1	−1	1		
E_1	2	1	−1	−2	0	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$
E_2	2	−1	−1	2	0	0		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$

Table 3 Character tables for the groups $\mathcal{D}_{nh} (n = 2, 3, 4, 5, 6)$.

$\mathcal{D}_{2h} = \mathcal{V}_h$	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
A_g	1	1	1	1	1	1	1	1		$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}$
B_{1g}	1	1	−1	−1	1	1	−1	−1	R_z	α_{xy}
B_{2g}	1	−1	1	−1	1	−1	1	−1	R_y	α_{zx}
B_{3g}	1	−1	−1	1	1	−1	−1	1	R_x	α_{yz}
A_u	1	1	1	1	−1	−1	−1	−1		
B_{1u}	1	1	−1	−1	−1	−1	1	1	T_z	
B_{2u}	1	−1	1	−1	−1	1	−1	1	T_y	
B_{3u}	1	−1	−1	1	−1	1	1	−1	T_x	

$\mathcal{D}_{3h}$	E	$2C_3$	$3C_2'$	σ_h	$2S_3$	$3\sigma_v$		
A_1'	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2'	1	1	−1	1	1	−1	R_z	
E'	2	−1	0	2	−1	0	(T_x, T_y)	$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
A_1''	1	1	1	−1	−1	−1		
A_2''	1	1	−1	−1	−1	1	T_z	
E''	2	−1	0	−2	1	0	(R_x, R_y)	$(\alpha_{yx}, \alpha_{zx})$

$\mathcal{D}_{4h}$	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_{2g}	1	1	1	−1	−1	1	1	1	−1	−1	R_z	
B_{1g}	1	−1	1	1	−1	1	−1	1	1	−1		$\alpha_{xx} - \alpha_{yy}$
B_{2g}	1	−1	1	−1	1	1	−1	1	−1	1		α_{xy}
E_g	2	0	−2	0	0	2	0	−2	0	0	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1		
A_{2u}	1	1	1	−1	−1	−1	−1	−1	1	1	T_z	
B_{1u}	1	−1	1	1	−1	−1	1	−1	−1	1		
B_{2u}	1	−1	1	−1	1	−1	1	−1	1	−1		
E_u	2	0	−2	0	0	−2	0	2	0	0	(T_x, T_y)	

$\mathcal{D}_{5h}$	E	$2C_5$	$2C_5^2$	$5C_2'$	σ_h	$2S_5$	$2S_5^3$	$5\sigma_v$		$\alpha = 72^\circ$
A_1'	1	1	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2'	1	1	1	−1	1	1	1	−1	R_z	
E_1'	2	$2\cos\alpha$	$2\cos 2\alpha$	0	2	$2\cos\alpha$	$2\cos 2\alpha$	0	(T_x, T_y)	
E_2'	2	$2\cos 2\alpha$	$2\cos\alpha$	0	2	$2\cos 2\alpha$	$2\cos\alpha$	0		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
A_1''	1	1	1	1	−1	−1	−1	−1		
A_2''	1	1	1	−1	−1	−1	−1	1	T_z	
E_1''	2	$2\cos\alpha$	$2\cos 2\alpha$	0	−2	$-2\cos\alpha$	$-2\cos 2\alpha$	0	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
E_2''	2	$2\cos 2\alpha$	$2\cos\alpha$	0	−2	$-2\cos 2\alpha$	$-2\cos\alpha$	0		

Table 3 (*Continued*).

$\mathscr{D}_{6h}$	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$	i	$2S_3$	$2S_6$	σ_h	$3\sigma_d$	$3\sigma_v$		
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
A_{2g}	1	1	1	1	−1	−1	1	1	1	1	−1	−1	R_z	
B_{1g}	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1		
B_{2g}	1	−1	1	−1	−1	1	1	−1	1	−1	−1	1		
E_{1g}	2	1	−1	−2	0	0	2	1	−1	−2	0	0	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
E_{2g}	2	−1	−1	2	0	0	2	−1	−1	2	0	0		$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy})$
A_{1u}	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1		
A_{2u}	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	T_z	
B_{1u}	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1		
B_{2u}	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1		
E_{1u}	2	1	−1	−2	0	0	−2	−1	1	2	0	0	(T_x, T_y)	
E_{2u}	2	−1	−1	2	0	0	−2	1	1	−2	0	0		

Table 4 Character tables for the groups $\mathscr{S}_n (n = 2, 4, 6, 8)$.

$\mathscr{S}_2 \equiv \mathscr{C}_i$	E	i		
A_g	1	1	R_x, R_y, R_z	$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}, \alpha_{xy}$ α_{yz}, α_{zx}
A_u	1	−1	T_x, T_y, T_z	

$\mathscr{S}_4$	E	S_4	C_2	S_4^3		
A	1	1	1	1	R_z	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
B	1	−1	1	−1	T_z	$\alpha_{xx}-\alpha_{yy}, \alpha_{xy}$
E	$\{$ 1	i	−1	$-i$ $\}$	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$
	1	$-i$	−1	i		

$\mathscr{S}_6$	E	C_3	C_3^2	i	S_6^5	S_6		$\varepsilon = exp(2\pi i/3)$
A_g	1	1	1	1	1	1	R_z	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
E_g	$\{$ 1	ε	$\varepsilon^\star$	1	ε	$\varepsilon^\star$ $\}$	(R_x, R_y)	$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy}), (\alpha_{yz}, \alpha_{zx})$
	1	$\varepsilon^\star$	ε	1	$\varepsilon^\star$	ε		
A_u	1	1	1	−1	−1	−1	T_z	
E_u	$\{$ 1	ε	$\varepsilon^\star$	−1	$-\varepsilon$	$-\varepsilon^\star$ $\}$	(T_x, T_y)	
	1	$\varepsilon^\star$	ε	−1	$-\varepsilon^\star$	$-\varepsilon$		

$\mathscr{S}_8$	E	S_8	C_4	S_8^3	C_2	S_8^5	C_4^3	S_8^7	$\varepsilon = exp(2\pi i/8)$	
A	1	1	1	1	1	1	1	1	R_z	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
B	1	−1	1	−1	1	−1	1	−1	T_z	
E_1	$\{$ 1	ε	i	$-\varepsilon^\star$	−1	$-\varepsilon$	$-i$	$\varepsilon^\star$ $\}$	(T_x, T_y)	
	1	$\varepsilon^\star$	$-i$	$-\varepsilon$	−1	$-\varepsilon^\star$	i	ε		
E_2	$\{$ 1	i	−1	$-i$	1	i	−1	$-i$ $\}$		$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy})$
	1	$-i$	−1	i	1	$-i$	−1	i		
E_3	$\{$ 1	$-\varepsilon^\star$	$-i$	ε	−1	$\varepsilon^\star$	i	$-\varepsilon$ $\}$	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
	1	$-\varepsilon$	i	$\varepsilon^\star$	−1	ε	$-i$	$-\varepsilon^\star$		

Table 5 Character tables for the groups $\mathcal{C}_{nh} (n = 1, 2, 3, 4, 5, 6)$.

$\mathcal{C}_{1h} \equiv \mathcal{C}_s$	E	σ_h		
A′	1	1	T_x, T_y, R_z	$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}, \alpha_{xy}$
A″	1	−1	T_z, R_x, R_y	α_{yz}, α_{zx}

$\mathcal{C}_{2h}$	E	C_2	i	σ_h		
A_g	1	1	1	1	R_z	$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}, \alpha_{xy}$
B_g	1	−1	1	−1	R_x, R_y	α_{yz}, α_{zx}
A_u	1	1	−1	−1	T_z	
B_u	1	−1	−1	1	T_x, T_y	

$\mathcal{C}_{3h}$	E	C_3	C_3^2	σ_h	S_3	S_3^5		$\varepsilon = exp(2\pi i/3)$
A′	1	1	1	1	1	1	R_z	$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
E′	1	ε	$\varepsilon^{\star}$	1	ε	$\varepsilon^{\star}$	(T_x, T_y)	$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
	1	$\varepsilon^{\star}$	ε	1	$\varepsilon^{\star}$	ε		
A″	1	1	1	−1	−1	−1	T_z	
E″	1	ε	$\varepsilon^{\star}$	−1	$-\varepsilon$	$-\varepsilon^{\star}$	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
	1	$\varepsilon^{\star}$	ε	−1	$-\varepsilon^{\star}$	$-\varepsilon$		

$\mathcal{C}_{4h}$	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	R_z	$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
B_g	1	−1	1	−1	1	−1	1	−1		$\alpha_{xx} - \alpha_{yy}, \alpha_{xy}$
E_g	1	i	−1	$-i$	1	i	−1	$-i$	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
	1	$-i$	−1	i	1	$-i$	−1	i		
A_u	1	1	1	1	−1	−1	−1	−1	T_z	
B_u	1	−1	1	−1	−1	1	−1	1		
E_u	1	i	−1	$-i$	−1	$-i$	1	i	(T_x, T_y)	
	1	$-i$	−1	i	−1	i	1	$-i$		

$\mathcal{C}_{5h}$	E	C_5	C_5^2	C_5^3	C_5^4	σ_h	S_5	S_5^7	S_5^3	S_5^9		$\varepsilon = exp(2\pi i/5)$
A′	1	1	1	1	1	1	1	1	1	1	R_z	$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
E_1'	1	ε	ε^2	$\varepsilon^{2\star}$	$\varepsilon^{\star}$	1	ε	ε^2	$\varepsilon^{2\star}$	$\varepsilon^{\star}$	(T_z, T_y)	
	1	$\varepsilon^{\star}$	$\varepsilon^{2\star}$	ε^2	ε	1	$\varepsilon^{\star}$	$\varepsilon^{2\star}$	ε^2	ε		
E_2'	1	ε^2	$\varepsilon^{\star}$	ε	$\varepsilon^{2\star}$	1	ε^2	$\varepsilon^{\star}$	ε	$\varepsilon^{2\star}$		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
	1	$\varepsilon^{2\star}$	ε	$\varepsilon^{\star}$	ε^2	1	$\varepsilon^{2\star}$	ε	$\varepsilon^{\star}$	ε^2		
A″	1	1	1	1	1	−1	−1	−1	−1	−1	T_z	
E_1''	1	ε	ε^2	$\varepsilon^{2\star}$	$\varepsilon^{\star}$	−1	$-\varepsilon$	$-\varepsilon^2$	$-\varepsilon^{2\star}$	$-\varepsilon^{\star}$	(R_x, R_y)	$(\alpha_{yz} + \alpha_{zx})$
	1	$\varepsilon^{\star}$	$\varepsilon^{2\star}$	ε^2	ε	−1	$-\varepsilon^{\star}$	$-\varepsilon^{2\star}$	$-\varepsilon^2$	$-\varepsilon$		
E_2''	1	ε^2	$\varepsilon^{\star}$	ε	$\varepsilon^{2\star}$	−1	$-\varepsilon^2$	$-\varepsilon^{\star}$	$-\varepsilon$	$-\varepsilon^{2\star}$		
	1	$\varepsilon^{2\star}$	ε	$\varepsilon^{\star}$	ε^2	−1	$-\varepsilon^{2\star}$	$-\varepsilon$	$-\varepsilon^{\star}$	$-\varepsilon^2$		

Table 5 (*Continued*).

$\mathcal{C}_{6h}$	E	C_6	C_3	C_2	C_3^2	C_6^5	i	S_3^5	S_6^5	σ_h	S_6	S_3		$\varepsilon = exp(2\pi i/6)$
A_g	1	1	1	1	1	1	1	1	1	1	1	1	R_z	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
B_g	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1		
E_{1g}	1	ε	$-\varepsilon^\star$	−1	$-\varepsilon$	$\varepsilon^\star$	1	ε	$-\varepsilon^\star$	−1	$-\varepsilon$	$\varepsilon^\star$	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
	1	$\varepsilon^\star$	$-\varepsilon$	−1	$-\varepsilon^\star$	ε	1	$\varepsilon^\star$	$-\varepsilon$	−1	$-\varepsilon^\star$	ε		
E_{2g}	1	$-\varepsilon^\star$	$-\varepsilon$	1	$-\varepsilon^\star$	$-\varepsilon$	1	$-\varepsilon^\star$	$-\varepsilon$	1	$-\varepsilon^\star$	$-\varepsilon$		$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy})$
	1	$-\varepsilon$	$-\varepsilon^\star$	1	$-\varepsilon$	$-\varepsilon^\star$	1	$-\varepsilon$	$-\varepsilon^\star$	1	$-\varepsilon$	$-\varepsilon^\star$		
A_u	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	T_z	
B_u	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1		
E_{1u}	1	ε	$-\varepsilon^\star$	−1	$-\varepsilon$	$\varepsilon^\star$	−1	$-\varepsilon$	$\varepsilon^\star$	1	ε	$-\varepsilon^\star$	(T_x, T_y)	
	1	$\varepsilon^\star$	$-\varepsilon$	−1	$-\varepsilon^\star$	ε	−1	$-\varepsilon^\star$	ε	1	$\varepsilon^\star$	$-\varepsilon$		
E_{2u}	1	$-\varepsilon^\star$	$-\varepsilon$	1	$-\varepsilon^\star$	$-\varepsilon$	−1	$\varepsilon^\star$	ε	−1	$\varepsilon^\star$	ε		
	1	$-\varepsilon$	$-\varepsilon^\star$	1	$-\varepsilon$	$-\varepsilon^\star$	−1	ε	$\varepsilon^\star$	−1	ε	$\varepsilon^\star$		

Table 6 Character tables for the groups $\mathcal{C}_{nv} (n = 2, 3, 4, 5, 6)$.

$\mathcal{C}_{2v}$	E	C_2	$\sigma_v(xz)$	$\sigma_v(yz)$		
A_1	1	1	1	1	T_z	$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}$
A_2	1	1	−1	−1	R_z	α_{xy}
B_1	1	−1	1	−1	T_x, R_y	α_{zx}
B_2	1	−1	−1	1	T_y, R_x	α_{yz}

$\mathcal{C}_{3v}$	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	T_z	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
A_2	1	1	−1	R_z	
E	2	−1	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy}), (\alpha_{yz}, \alpha_{zx})$

$\mathcal{C}_{4v}$	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	T_x	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	−1	−1	R_z	
B_1	1	−1	1	1	−1		$\alpha_{xx}-\alpha_{yy}$
B_2	1	−1	1	−1	1		α_{xy}
E	2	0	−2	0	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$

$\mathcal{C}_{5v}$	E	$2C_5$	$2C_5^2$	$5\sigma_v$		$\alpha = 72°$
A_1	1	1	1	1	T_x	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	−1	R_z	
E_1	2	$2\cos\alpha$	$2\cos 2\alpha$	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$
E_2	2	$2\cos 2\alpha$	$2\cos\alpha$	0		$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy})$

$\mathcal{C}_{6v}$	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	T_z	$\alpha_{xx}+\alpha_{yy}, \alpha_{xy}$
A_2	1	1	1	1	−1	−1	R_z	
B_1	1	−1	1	−1	1	−1		
B_2	1	−1	1	−1	−1	1		
E_1	2	1	−1	−2	0	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$
E_2	2	−1	−1	2	0	0		$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy})$

Table 7 Character tables for the groups, $\mathcal{D}_{nd}(n = 2, 3, 4, 5, 6)$.

$\mathcal{D}_{2d} = \mathcal{V}_d$	E	$2S_4$	C_2	$2C_2'$	$2\sigma_d$		
A_1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	−1	−1	R_z	
B_1	1	−1	1	1	−1		$\alpha_{xx} - \alpha_{yy}$
B_2	1	−1	1	−1	1	T_z	α_{xy}
E	2	0	−2	0	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$

$\mathcal{D}_{3d}$	E	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_d$		
A_{1g}	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_{2g}	1	1	−1	1	1	−1	R_z	
E_g	2	−1	0	2	−1	0	(R_x, R_y)	$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy}), (\alpha_{yz}, \alpha_{zx})$
A_{1u}	1	1	1	−1	−1	−1		
A_{2u}	1	1	−1	−1	−1	1	T_z	
E_u	2	−1	0	−2	1	0	(T_x, T_y)	

$\mathcal{D}_{4d}$	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C_2'$	$4\sigma_d$		
A_1	1	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	1	1	−1	−1	R_z	
B_1	1	−1	1	−1	1	1	−1		
B_2	1	−1	1	−1	1	−1	1	T_z	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0	(T_x, T_y)	
E_2	2	0	−2	0	2	0	0		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$

$\mathcal{D}_{5d}$	E	$2C_5$	$2C_5^2$	$5C_2$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$		$\alpha = 72°$
A_{1g}	1	1	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_{2g}	1	1	1	−1	1	1	1	−1	R_z	
E_{1g}	2	$2\cos\alpha$	$2\cos 2\alpha$	0	2	$2\cos\alpha$	$2\cos 2\alpha$	0	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$
E_{2g}	2	$2\cos 2\alpha$	$2\cos\alpha$	0	2	$2\cos 2\alpha$	$2\cos\alpha$	0		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
A_{1u}	1	1	1	1	−1	−1	−1	−1		
A_{2u}	1	1	1	−1	−1	−1	−1	1	T_z	
E_{1u}	2	$2\cos\alpha$	$2\cos 2\alpha$	0	−2	$-2\cos\alpha$	$-2\cos 2\alpha$	0	(T_x, T_y)	
E_{2u}	2	$2\cos 2\alpha$	$2\cos\alpha$	0	−2	$-2\cos 2\alpha$	$-2\cos\alpha$	0		

$\mathcal{D}_{6d}$	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^5$	C_2	$6C_2'$	$6\sigma_d$		
A_1	1	1	1	1	1	1	1	1	1		$\alpha_{xx} + \alpha_{yy}, \alpha_{zz}$
A_2	1	1	1	1	1	1	1	−1	−1	R_z	
B_1	1	−1	1	−1	1	−1	1	1	−1		
B_2	1	−1	1	−1	1	−1	1	−1	1	T_z	
E_1	2	$\sqrt{3}$	1	0	−1	$-\sqrt{3}$	−2	0	0	(T_x, T_y)	
E_2	2	1	−1	−2	−1	1	2	0	0		$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
E_3	2	0	−2	0	2	0	−2	0	0		
E_4	2	−1	−1	2	−1	−1	2	0	0		
E_5	2	$-\sqrt{3}$	1	0	−1	$\sqrt{3}$	−2	0	0	(R_x, R_y)	$(\alpha_{yz}, \alpha_{zx})$

Table 8 Character tables for the cubic groups.

$\mathcal{T}$	E	$4C_3$	$4C_3^2$	$3C_2$		$\varepsilon = exp(2\pi i/3)$
A	1	1	1	1		$\alpha_{xx}+\alpha_{yy}+\alpha_{zz}$
E	1	ε	$\varepsilon^\star$	1		$(\alpha_{xx}+\alpha_{yy}-2\alpha_{zz}, \alpha_{xx}-\alpha_{yy})$
	1	$\varepsilon^\star$	ε	1		
T ≡ F	3	0	0	−1	**T, R**	$(\alpha_{xy}, \alpha_{yz}, \alpha_{zx})$

$\mathcal{T}_h$	E	$4C_3$	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_6^5$	3σ		$\varepsilon = exp(2\pi i/3)$
A_g	1	1	1	1	1	1	1	1		$\alpha_{xx}+\alpha_{yy}+\alpha_{zz}$
E_g	1	ε	$\varepsilon^\star$	1	1	$\varepsilon^\star$	ε	1		$(\alpha_{xx}+\alpha_{yy}-2\alpha_{zz},$
	1	$\varepsilon^\star$	ε	1	1	ε	$\varepsilon^\star$	1		$\alpha_{xx}-\alpha_{yy})$
$T_g \equiv F_g$	3	0	0	−1	3	0	0	−1	**R**	$(\alpha_{xy}, \alpha_{yz}, \alpha_{zx})$
A_u	1	1	1	1	−1	−1	−1	−1		
E_u	1	ε	$\varepsilon^\star$	1	−1	$-\varepsilon^\star$	$-\varepsilon$	−1		
	1	$\varepsilon^\star$	ε	1	−1	$-\varepsilon$	$-\varepsilon^\star$	−1		
$T_u \equiv F_u$	3	0	0	−1	−3	0	0	1	**T**	

$\mathcal{T}_d$	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
A_1	1	1	1	1	1		$\alpha_{xx}+\alpha_{yy}+\alpha_{zz}$
A_2	1	1	1	−1	−1		
E	2	−1	2	0	0		$(\alpha_{xx}+\alpha_{yy}-2\alpha_{zz}, \alpha_{xx}-\alpha_{yy})$
$T_1 \equiv F_1$	3	0	−1	1	−1	**R**	
$T_2 \equiv F_2$	3	0	−1	−1	1	**T**	$(\alpha_{xy}, \alpha_{yz}, \alpha_{zx})$

$\mathcal{O}_h$	E	$8C_3$	$3C_2$	$6C_4$	$6C_2'$	i	$8S_6$	$3\sigma_h$	$6S_4$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$\alpha_{xx}+\alpha_{yy}+\alpha_{zz}$
A_{2g}	1	1	1	−1	−1	1	1	1	−1	−1		
E_g	2	−1	2	0	0	2	−1	2	0	0		$(\alpha_{xx}+\alpha_{yy}-2\alpha_{zz},$ $\alpha_{xx}-\alpha_{yy})$
$T_{1g} \equiv F_{1g}$	3	0	−1	1	−1	3	0	−1	1	−1	**R**	
$T_{2g} \equiv F_{2g}$	3	0	−1	−1	1	3	0	−1	−1	1		$(\alpha_{xy}, \alpha_{yz}, \alpha_{zx})$
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1		
A_{2u}	1	1	1	−1	−1	−1	−1	−1	1	1		
E_u	2	−1	2	0	0	−2	1	−2	0	0		
$T_{1u} \equiv F_{1u}$	3	0	−1	1	−1	−3	0	1	−1	1	**T**	
$T_{2u} \equiv F_{2u}$	3	0	−1	−1	1	−3	0	1	1	−1		

Table 9 Character tables for the icosahedral groups.

$\mathscr{I}$	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$		
A	1	1	1	1	1		$\alpha_{zz}+\alpha_{yy}+\alpha_{zz}$
$T_1 \equiv F_1$	3	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$	0	−1	**T**, **R**	
$T_2 \equiv F_2$	3	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$	0	−1		
G	4	−1	−1	1	0		
H	5	0	0	−1	1		$(\alpha_{xx}+\alpha_{yy}-2\alpha_{zz}, \alpha_{xx}-\alpha_{yy}, \alpha_{xy}, \alpha_{yz}, \alpha_{zx})$

$\mathscr{I}_h$	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	i	$12S_{10}$	$12S_{10}^3$	$20S_6$	15σ		
A_g	1	1	1	1	1	1	1	1	1	1		$\alpha_{xx}+\alpha_{yy}+\alpha_{zz}$
$T_{1g} \equiv F_{1g}$	3	$\frac{1}{2}(1+\sqrt{5})$	$\frac{1}{2}(1-\sqrt{5})$	0	−1	3	$\frac{1}{2}(1-\sqrt{5})$	$\frac{1}{2}(1+\sqrt{5})$	0	−1	**R**	
$T_{2g} \equiv F_{2g}$	3	$\frac{1}{2}(1-\sqrt{5})$	$\frac{1}{2}(1+\sqrt{5})$	0	−1	3	$\frac{1}{2}(1+\sqrt{5})$	$\frac{1}{2}(1-\sqrt{5})$	0	−1		
G_g	4	−1	−1	1	0	4	−1	−1	1	0		
H_g	5	0	0	−1	1	5	0	0	−1	1		$(\alpha_{xx}+\alpha_{yy}-2\alpha_{zz}$, $\alpha_{xx}-\alpha_{yy}, \alpha_{xy}$, $\alpha_{yz}, \alpha_{zx})$
A_u	1	1	1	1	1	−1	−1	−1	−1	−1		
$T_{1u} \equiv F_{1u}$	3	$\frac{1}{2}(1+\sqrt{5})$	$\frac{1}{2}(1-\sqrt{5})$	0	−1	−3	$-\frac{1}{2}(1-\sqrt{5})$	$-\frac{1}{2}(1+\sqrt{5})$	0	1	**T**	
$T_{2u} \equiv F_{2u}$	3	$\frac{1}{2}(1-\sqrt{5})$	$\frac{1}{2}(1+\sqrt{5})$	0	−1	−3	$-\frac{1}{2}(1+\sqrt{5})$	$-\frac{1}{2}(1-\sqrt{5})$	0	1		
G_u	4	−1	−1	1	0	−4	1	1	−1	0		
H_u	5	0	0	−1	1	−5	0	0	1	−1		

Table 10 Character tables for the infinite groups of linear structures.

$C_{\infty v}$	E	$2C_{\infty}^{\phi}$	$\cdots$	$\infty\sigma_v$		
$A_1 \equiv \Sigma^+$	1	1	$\cdots$	1	T_z	$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
$A_2 \equiv \Sigma^-$	1	1	$\cdots$	-1	R_z	
$E_1 \equiv \Pi$	2	$2\cos\phi$	$\cdots$	0	$(T_x, T_y), (R_x, R_y)$	$(\alpha_{yz}, \alpha_{zx})$
$E_2 \equiv \Delta$	2	$2\cos 2\phi$	$\cdots$	0		$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy})$
$E_3 \equiv \Phi$	2	$2\cos 3\phi$	$\cdots$	0		
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$		

$D_{\infty h}$	E	$2C_{\infty}^{\phi}$	$\cdots$	$\infty\sigma_v$	i	$2S_{\infty}^{\phi}$	$\cdots$	∞C_2		
Σ_g^+	1	1	$\cdots$	1	1	1	$\cdots$	1		$\alpha_{xx}+\alpha_{yy}, \alpha_{zz}$
Σ_g^-	1	1	$\cdots$	-1	1	1	$\cdots$	-1	R_z	
Π_g	2	$2\cos\phi$	$\cdots$	0	2	$-2\cos\phi$	$\cdots$	0	(R_z, R_y)	$(\alpha_{yz}, \alpha_{zx})$
Δ_g	2	$2\cos 2\phi$	$\cdots$	0	2	$2\cos 2\phi$	$\cdots$	0		$(\alpha_{xx}-\alpha_{yy}, \alpha_{xy})$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$		
Σ_u^+	1	1	$\cdots$	1	-1	-1	$\cdots$	-1	T_z	
Σ_u^-	1	1	$\cdots$	-1	-1	-1	$\cdots$	1		
Π_u	2	$2\cos\phi$	$\cdots$	0	-2	$2\cos\phi$	$\cdots$	0	(T_x, T_y)	
Δ_u	2	$2\cos 2\phi$	$\cdots$	0	-2	$-2\cos 2\phi$	$\cdots$	0		
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$		