Appendix IX: Matrix Elements for the Harmonic Oscillator

Some of the more useful matrix elements for the harmonic oscillator are presented in the following table. They are given as functions of the dimensionless quantities $\xi = 2\pi x\sqrt{\nu m/h}$ and $\sigma = 2\varepsilon/h\nu$, as defined in Section 6.2.

$\langle v\|\xi\|v+1\rangle = \sqrt{\frac{1}{2}(v+1)}$	$\langle v\|\xi^3\|v-1\rangle = \frac{3}{2}\sqrt{v^3/2}$
$\langle v\|\xi\|v-1\rangle = \sqrt{\frac{1}{2}v}$	$\langle v\|\xi^3\|v-3\rangle = \frac{1}{2}\sqrt{v(v-1)(v-2)/2}$
$\langle v\|\xi\|v'\rangle = 0$, if $v' \neq v \pm 1$	$\langle v\|\xi^3\|v'\rangle = 0$, if $v' \neq v \pm 1$ or $v' \neq v \pm 3$
$\langle v\|\xi^2\|v+2\rangle = \frac{1}{2}\sqrt{(v+1)(v+2)}$	$\langle v\|\xi^4\|v+4\rangle = \frac{1}{4}\sqrt{(v+1)(v+2)(v+3)(v+4)/2}$
$\langle v\|\xi^2\|v\rangle = v + \frac{1}{2}$	$\langle v\|\xi^4\|v+2\rangle = \frac{1}{2}(2v+3)\sqrt{(v+1)(v+2)}$
$\langle v\|\xi^2\|v-2\rangle = \frac{1}{2}\sqrt{v(v-1)}$	$\langle v\|\xi^4\|v\rangle = \frac{3}{4}(2v^2+2v+1)$
$\langle v\|\xi^2\|v'\rangle = 0$, if $v' \neq v \pm 2$	$\langle v\|\xi^4\|v-2\rangle = \frac{1}{2}(2v-1)\sqrt{v(v-1)}$
$\langle v\|\xi^3\|v+3\rangle = \frac{1}{2}\sqrt{(v+1)(v+2)(v+3)/2}$	$\langle v\|\xi^4\|v-4\rangle = \frac{1}{4}\sqrt{v(v-1)(v-2)(v-3)/2}$
$\langle v\|\xi^3\|v+1\rangle = \frac{3}{2}\sqrt{[(v+1)/2]^3}$	$\langle v\|\xi^4\|v'\rangle = 0$, if $v' \neq v, v \pm 2$ or $v \pm 4$